Lesson 3-8: Rate of Change

Rate of Change: A ratio that describes, on average, how much one quantity changes with respect to a change in another quantity.

general formula: rate of change = $\frac{\text{change in y}}{\text{change in x}}$ $\frac{y_2 - y_1}{x_2 - x_1}$ = slope

Example 1: Use the table to find the rate of change. Then explain its meaning.

Number of computer games	Total cost ($)
x	**y**
2	78
4	156
6	234

rate of change = $\frac{\text{change in y} \rightarrow \text{dollars}}{\text{change in x} \rightarrow \text{games}}$ $\frac{\text{difference in cost}}{\text{difference in \# of games}}$

= $\frac{\text{change in cost}}{\text{change in \# of games}}$

x_1, y_1 $x_2\ y_2$
(2, 78) and (6, 234)

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{234 - 78}{6 - 2} = \frac{156}{4} = \frac{39}{1}$

= 39 This means the cost per game is \$39.

Example 2: The table shows how the tiled surface area changes with the number of floor tiles. Find the rate of change. Explain the meaning of the rate of change.

Number of floor tiles	Area of tiles surface (in^2)
X	**Y**
3	48
6	96
9	144

rate of change = $\frac{y_2 - y_1}{x_2 - x_1}$

$x_1\ y_1$ $x_2\ y_2$
(3, 48) and (9, 144)

$\frac{144 - 48}{9 - 3} = \frac{96}{6} = \frac{16}{1}$ $\frac{\text{surface area}}{\text{tile}}$

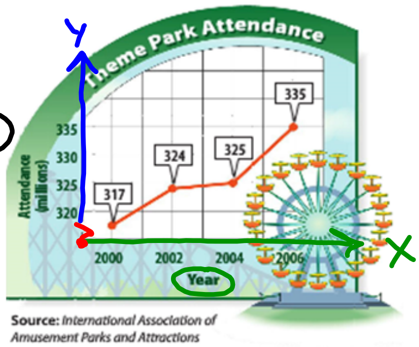
rate of change = 16 in^2 per tile

So far you have seen rates of change that are *constant*. Many real world situations involve rates of change that are *not* constant.

Example 3: The graph shows the number of people who visited U.S. theme parks in the recent years.

rate of change = $\frac{y_2-y_1}{x_2-x_1}$

a. Find the rates of change for 2000-2002 and 2002-2004.



2000–2002	2002–2004
(2000,317),(2002,324)	(2002,324), (2004,325)
x_1 y_1 x_2 y_2	x_1 y_1 x_2 y_2
$\frac{324-317}{2002-2000} = \frac{7}{2}$	$\frac{325-324}{2004-2002} = \frac{1}{2}$
3.5 million people per yr.	0.5 million people per yr.

b. Explain the meaning of the rate of change in each case.

For 2000-2002, park attendance increased by 3.5 million people per year.

For 2002-2004, park attendance increased by 0.5 million people per year.

c. How are the different rates of change shown on the graph?

The slope for 2000-2002 is STEEPER than the slope for 2002-2004.

d. Without calculating, find the 2 year period that has the least rate of change. Then calculate to verify your answer.

2002-2004

2000-2002:	2002-2004:	2004-2006:
+3.5 million people per yr.	+0.5 million people per yr. (least)	(2004, 325), (2006,335)
		x_1 y_1 x_2 y_2
		$\frac{335-325}{2006-2004} = \frac{10}{2} = 5$
		+5 million people per yr. (most)

LINEAR vs. NONLINEAR FUNCTIONS

For a rate of change to be linear, the change in x-values must be constant and the change in y-values must be constant (but their rates of change don't have to be identical to each other).

Example 4:

Determine whether each function is linear. Explain.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

a.

x	y
1	−6
4	−8
7	−10
10	−12
13	−14

x: +3, +3, +3, +3 — constant

y: −2, −2, −2, −2 — constant

This is a linear function. $\rightarrow m = \frac{-2}{3}$

b.

x	y
−3	10
−1	12
1	16
3	18
5	22

x: +2, +2, +2, +2 — constant

y: +2, +4, +2, +4 — not constant

This is NOT a linear function.

Try these on your own:

Determine whether each function is linear. Explain.

x	y
−3	11
−2	15
−1	19
1	23
2	27

x: +1, +1, +2, +1 — not constant

y: +4, +4, +4, +4 — constant

not linear

x	y
12	−4
9	1
6	6
3	11
0	16

x: −3, −3, −3, −3

y: +5, +5, +5, +5

linear

$m = \frac{5}{-3}$